LETTER TO THE EDITOR

Total spectrum of photon emission by an ultra-relativistic positron channelling in a periodically bent crystal. ‡

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Abstract. We present the results of numerical calculations of the channelling and undulator radiation generated by an ultra-relativistic positron channelling along a crystal plane, which is periodically bent. The bending might be due either to the propagation of a transverse acoustic wave through the crystal, or due to the static strain as it occurs in superlattices. The periodically bent crystal serves as an undulator. We investigate the dependence of the intensities of both the ordinary channelling and the undulator radiations on the parameters of the periodically bent channel with simultaneous account for the dechannelling effect of the positrons. We demonstrate that there is a range of parameters in which the undulator radiation dominates over the channelling one and the characteristic frequencies of both types of radiation are well separated. This result is important, because the undulator radiation can be used to create a tunable source of X-ray and γ-radiation.

In this Letter we present for the first time the results of calculations of the total spectrum of emitted photons accompanying the channelling process of ultra-relativistic

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charged particles through a crystal which is periodically bent. The present consideration is a further step in investigating new phenomenon which was described recently in [1, 2] and was called Acoustically Induced Radiation (AIR). It was noted that the periodic pattern of crystal bendings (which can be achieved either through propagation of a transverse acoustic wave, AW, or by using static periodically strained crystalline structures [2, 3]) gives rise to a new mechanism of electromagnetic emission of the undulator type, in addition to a well-known ordinary channelling radiation [4].

Without any loss of generality further in this Letter we consider the case of a dynamic periodic bending of a crystal by means of transverse AW, as it is illustrated in figure 1. Under the action of the transverse AW propagating along the z-direction, which defines the center line of initially straight channel (not plotted in the figure) the channel becomes periodically bent. Provided certain conditions are fulfilled [2] the beam of particles, which enters the crystal at a small incident angle with respect to the curved crystallographic plane, will penetrate through the crystal following the bendings of its channel. This results in transverse oscillations of the beam particles (*additional* to the oscillations inside the channel due to the action of the interplanar force). These oscillations become an effective source of spontaneous radiation of undulator type due to the constructive interference of the photons emitted from similar parts of the trajectory. It was demonstrated [2] that the system "ultra-relativistic charged particle + periodically bent crystal" serves as a new type of undulator, and, consequently, as a new source of undulator radiation of high intensity, monochromaticity and of a particular pattern of the angular-frequency distribution.

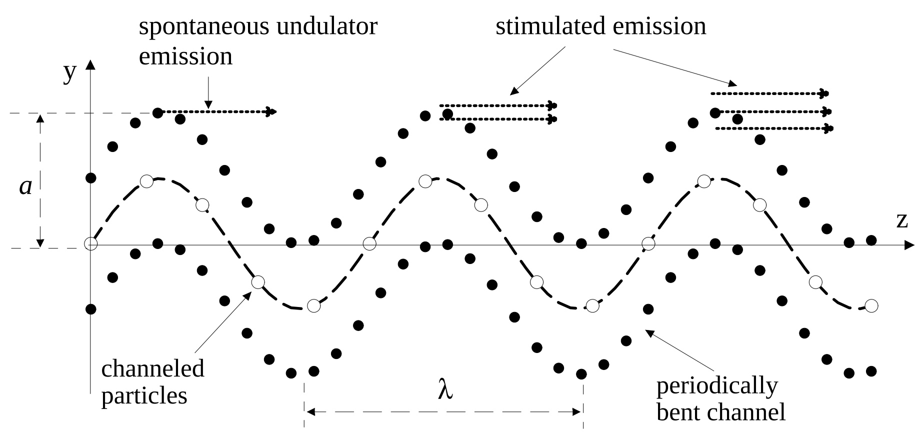

Figure 1. Schematic representation of spontaneous and stimulated AIR in a channel bent by the transverse AW. The y- and z-scales are incompatible.

As it was pointed out in [2] this scheme leads, in addition to the spontaneous radiation, to a possibility to generate stimulated emission, similar to the one known for a free electron laser [6] in which the periodicity of a trajectory of an ultra-relativistic projectile is achieved by applying spatially periodic magnetic field. In connection with the stimulated AIR it was noted in [2] that to achieve noticeable degree of amplification one has to operate with a positron bunch of a high volume density, which, nevertheless, turned out to be achievable in modern accelerators as discussed in [5].

The main subject of our previous studies [1, 2] was the characteristics of the undulator AIR itself. Due to this reason we primarily investigate the case of a high-amplitude AW, $a \gg d$ with d standing for the interplanar spacing, propagating through the crystal. Less attention, except for some qualitative estimates, has been paid to the detailed investigation of the mutual influence of two types of radiation, the ordinary channelling radiation and the AIR, in forming the total spectrum of the radiation emitted by a channeled positron. This topic is addressed in the present Letter where we report our first quantitative results on numerical calculation of the spectrum of the emitted photons with both mechanisms taken into account simultaneously. We demonstrate that there are ranges of (i) the parameters of AW, which are the amplitude, a, the wavelength λ, and the sound velocity V, (ii) the energies of projectile positron, ε, (iii) the crystal parameters, which include the length of a crystal, the constituent atoms and the types of crystallographic planes, inside which

- the characteristic frequencies of the AIR and the ordinary channelling radiation are well separated,
- the intensity of AIR is essentially higher than that of the ordinary channelling radiation,
- the radiative spectrum is stable towards the total losses of the particle (in the case of a positron it is primarily the radiative ones)

These items, except the last one which was considered in [7], are discussed below in the Letter.

To conclude the introductory part we mention that in the past the problem of evaluating the total spectrum of radiation formed in a bent crystal was considered in several publications [8, 9] in the case of a projectile channelling in a crystal bent with a constant curvature radius. In our Letter for the first we investigate the problem for the periodically bent channel. In full the results of our research will be published elsewhere [10]. Below we present the essential points.

An adequate approach to the problem of the radiation emission by an ultra-relativistic particle moving in an external field was developed by Baier and Katkov in the late 1960s [11] and was called "operator quasi-classical method" by the authors. The details of that formalism can be found in [12, 13]. The advantage of this method is that it allows to use the classical trajectory for the particle in an external field and, simultaneously, it takes into account the effect of the radiative recoil.

For particles with spin $s = 1/2$ the energy radiated into a given direction $\mathbf{n}$ summed over the polarizations of the photon and the projectile is given by (the CGS system is adopted throughout the paper)

$$\mathrm{d}E \equiv \frac{\mathrm{d}E}{\hbar \mathrm{d}\omega\, \mathrm{d}\Omega_{\mathbf{n}}} = \frac{\alpha\, \omega^2}{4\pi^2} \int_0^\tau \mathrm{d}t_1 \int_0^\tau \mathrm{d}t_2\ \mathrm{e}^{\mathrm{i}\omega' \varphi(t_1,t_2)}\ f(t_1,t_2), \tag{1}$$

where $\alpha \approx 1/137$ is the fine structure constant, $\varphi(t_1,t_2) = t_1 - t_2 - \mathbf{n}\cdot(\mathbf{r}(t_1) - \mathbf{r}(t_2))/c$, and $f(t_1,t_2) = \left\{\left[1 + (1+u)^2\right](\mathbf{v}(t_1)\mathbf{v}(t_2)/c^2 - 1) + u^2\gamma^{-2}\right\}/2$, where c is the velocity of light, $\gamma = \varepsilon/m$ is the relativistic factor and $u = \hbar\omega/(\varepsilon - \hbar\omega)$.

The main advantage of the the quasi-classical approach is that to calculate the angular-spectral distribution of the radiation one only needs to know the time dependencies of classical radius-vector $\mathbf{r}(t)$ and the velocity $\mathbf{v}(t)$ of projectile. In connection with a positron channelling through a periodically bent crystal the above relations impose, if applied directly, some restrictions on the projectile energy, on the parameters of the crystal (channel) and the AW.

The channelling process in a bent crystal takes place if the centrifugal force in the channel is less than the maximal force due to the interplanar field [14]. For a periodically bent crystal the maximal centrifugal force is equal to $m\gamma v^2/R_{\min}$, with $v \approx c$ and $R_{\min}$ being a minimum curvature radius of the bent channel. Hence, the following condition must be fulfilled [1, 2]

$$m\gamma c^2/R_{\min} < U'_{\max}. \tag{2}$$

where $U'_{\max}$ stands for the maximum gradient of the interplanar field. For an acoustically bent channel $R_{\min} = (\lambda/2\pi)^2/a$, therefore, the inequality (2) bounds together the characteristics of the crystal, $U'_{\max}$, the AW amplitude and the wavelength, and the relativistic factor γ. A comprehensive study of the allowed ranges of all parameters involved in (2) was carried out in [1, 2].

Second requirement which has to be fulfilled to make eq. (1) directly applicable to calculating the spectrum accompanying the channelling process concerns the upper limit of integration τ, which is related to the crystal length L through $L = c\tau$, and which, for given value of the AW wavelength, defines the number of the undulator periods as $N = L/\lambda$. The AIR acquires specific features of the undulator-type radiation (such as the monochromaticity and particular pattern of the angular-frequency distribution, see e.g. [12]) provided $N \gg 1$. The length of a crystal is subject to a physical condition that the positron bunch stays inside the channel when penetrating through the crystal on the scale of L. In reality, the parasitic effect, the dechannelling leads to a decrease in the volume density of the channeled particles $n(z)$ with penetration distance z, and roughly satisfies the exponential decay law for both straight and bent channels (see [15, 16]), $n(z) = n_0 \exp(-z/L_d)$, where n_0 is the volume density t the entrance, and L_d is the dechannelling length, which for given crystal and channel depends on a positron energy and on the curvature radius. For a periodically bent crystal $L_d(\gamma, R)$ can be estimated as [2, 16, 17]

$$L_d(\gamma, R) = (1 - R_c/R_{\min})^2 \, \frac{256}{9\pi^2} \, \frac{a_{\rm TF}}{r_{\rm cl}} \, \frac{d}{L_c} \, \gamma \tag{3}$$

where $r_{\rm cl}$ is the classical radius of an electron, $a_{\rm TF} = 0.8853 Z^{-1/3} a_0$ (a_0 is the Bohr radius) and $I = 16 Z^{0.9}$ eV are, respectively, the Thomas-Fermi radius and ionization potential of the crystal atoms, Z is the atomic number, d is the interplanar distance in the lattice. The quantity $R_c = \varepsilon/U'_{\max}$ is the critical (minimal) radius consistent with the channelling condition in a bent crystal (2). The quantity $L_c = \ln\left(\sqrt{2\gamma}\, mc^2/I\right) - 23/24$ is the Coulomb logarithm characterizing the ionization losses of an ultra-relativistic positron in amorphous media with account for the density effect (see e.g. [17, 18]).

The AIR will have a pronounced pattern of undulator-type radiation provided the number of the periods is large on the scale of L_d. Combined with (3) this condition leads to another restriction on the parameters involved

$$N = (1 - R_c/R_{\min})^2 \, \frac{256}{9\pi^2} \, \frac{a_{\rm TF}}{r_{\rm cl}} \, \frac{d}{L_c} \, \frac{\gamma}{\lambda} \gg 1 \tag{4}$$

where λ can be related to the AW frequency ν through $\lambda = V/\nu$ with V standing for the sound velocity.

Figure 2 illustrates the restrictions which are imposed on the values of a and ν by inequalities (2) and (4) in the case of $\varepsilon = 0.5$ GeV planar channelling in Si along (110) crystallographic planes. The diagonal straight lines correspond to various values (as indicated) of the parameter $C = \varepsilon/(R_{\min} U'_{\max}) \leq 1$ consistent with the channelling condition (2). The curved lines correspond to various values (as indicated) of the number of undulator periods N related to the dechannelling length L_d through eq. (4). The horizontal lines mark the values of the AW amplitude equal to d (with $d = 1.92\,\mathring{A}$ being the (110) interplanar distance in Si) and to $10\,d$. The vertical line marks the value $\nu = 200$ MHz of the AW frequency for which the spectra (1) were calculated. We used the value $V = 4.67 \times 10^5$ cm/s for the velocity of sound in Si (this value was obtained by using the data from [19]). Thus, the AW wave-length used in our calculations equals to $\lambda = 2.33 \times 10^{-3}$ cm.

The calculated spectra of the radiation emitted in the forward direction (with respect to the z-axis, see figure 1) for photon energies from 45 keV to 1.5 MeV are presented in figures 3. The details of the analytical evaluation of the right-hand side of (1) and its numerical implementation as well as more extended results of the calculation will be published soon [10]. Here we briefly sketch the numeric procedure and present a discussion of the exhibited results. The AW frequency, the number of undulator periods and crystal length were fixed at $\nu = 200$ MHz, $N = 15$ and $L = N\,\lambda = 3.5 \times 10^{-2}$ cm. The ratio a/d of the AW amplitude to the interplanar spacing was varied within the interval $a/d = 0 \dots 10$. The case $a/d = 0$ corresponds to the straight channel.

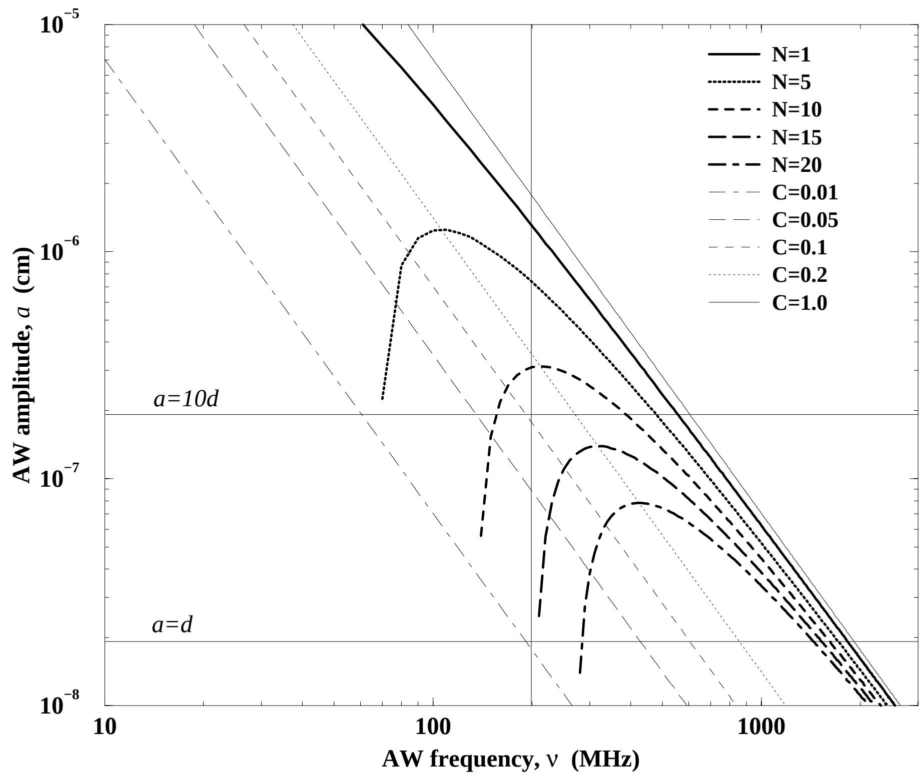

Figure 2. The ranges of the allowed parameters of the AW, a versus ν, consistent with the conditions (2) and (4) for $\varepsilon = 0.5$ GeV ($\gamma \approx 10^3$) positron channeling in Si along (110) crystallographic planes. See explanations in the text for the meaning of various lines.

To evaluate the spectral distribution (1) the following procedure was adopted.

Firstly, the spectrum was calculated for individual trajectories. These were obtained by solving the relativistic equations of motion with both the interplanar and the centrifugal potentials taken into account. We used the continuum approximation [20] to describe the projectile positron − lattice atoms interaction. Within these scheme we considered two frequently used [15] analytic forms for the interplanar potential, the harmonic and the Molière potentials calculated at the temperature $T = 150$ K to account for the thermal vibrations of the lattice atoms. For each a/d value by changing the initial values of the entrance coordinate $y^{(0)}$ and the initial velocities along the y-axis (see figure 1) we left only those trajectories which corresponded to the case of stable channelling through the whole crystal length L. We call a trajectory as a "stable" one if moving along it the particle does not approach crystalline planes at a distance less than the Thomas-Fermi radius $a_{\rm TF}$ ($a_{\rm TF} = 0.194$ Å for a Si atom). This allowed us to totally disregard, at least on the scale $L \sim L_d$, the random scattering of a projectile by lattice electrons (see e.g. [15, 16]). Thus, for each a/d value, we defined the ranges of the initial coordinates $y^{(0)} \in [-d/2 + a_{\rm TF}, d/2 - a_{\rm TF}]$ and the velocities $v_y^{(0)}$, and,

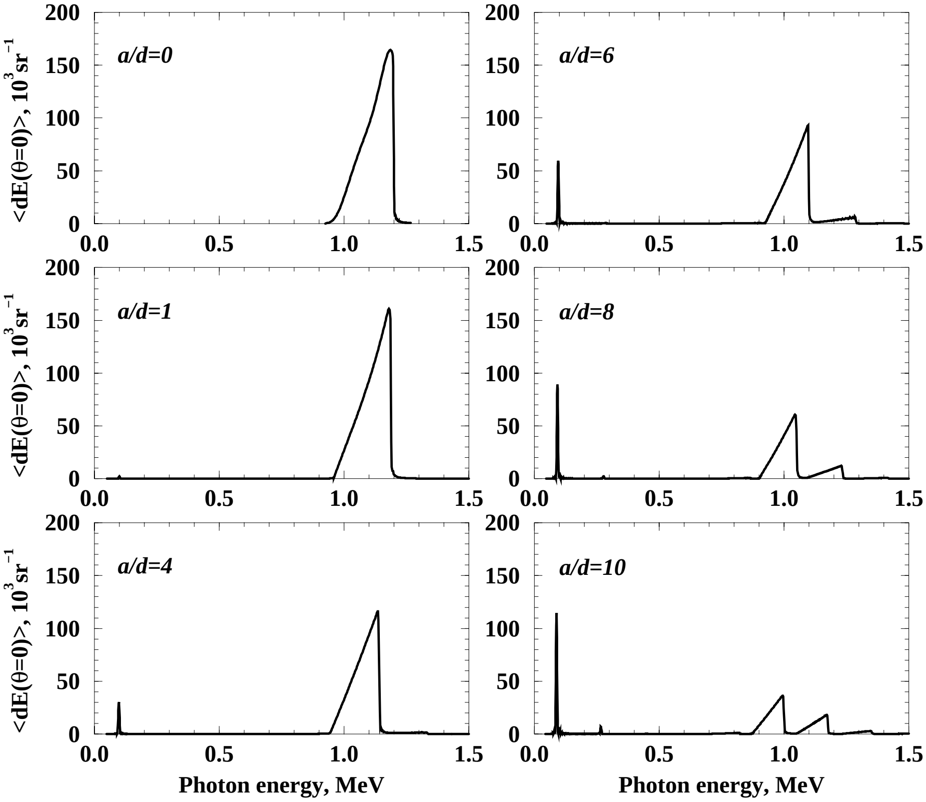

Figure 3. Spectral distribution (5) of the total radiation emitted in the forward direction ($\mathbf{n}\,\|z$) for $\varepsilon = 0.5$ GeV ($\gamma \approx 10^3$) positron channeling in Si along (110) crystallographic planes calculated at different a/d ratios as indicated. The AW frequency is fixed at $\nu = 200$ MHz, the crystal length is $L = 3.5 \times 10^{-2}$ cm.

correspondingly, the initial phase volume $\Phi^{(0)}(a/d) = \oint p_y^{(0)} \mathrm{d}y^{(0)}$ (where $p_y^{(0)}$ stands for initial transverse momentum) for which the corresponding classical trajectories are stable when channelling through the whole crystal length L. Then, discretizing the found initial phase volume $\Phi^{(0)}(a/d)$ by choosing $N_{y^{(0)}} \times N_{v_y^{(0)}}$ points $(y^{(0)}, p_y^{(0)}) \in \Phi^{(0)}(a/d)$, the individual spectra $\mathrm{d}E(y^{(0)}, p_y^{(0)})$ were calculated for each pair of the initial coordinate and velocity. Finally, for each a/d value we evaluated the averaged spectrum defined as follows:

$$\langle \mathrm{d}E \rangle = \frac{1}{\Phi^{(0)}(a/d = 0)} \oint_{\Phi^{(0)}(a/d)} \left[\mathrm{d}E(y^{(0)}, p_y^{(0)})\right] p_y^{(0)} \mathrm{d}y^{(0)} \qquad (5)$$

Here, the integration is carried out over the phase volume $\Phi^{(0)}(a/d)$, and the integral is scaled by the phase volume $\Phi^{(0)}(a/d = 0)$ of stable trajectories in straight channel. The ratio $\Phi^{(0)}(a/d)/\Phi^{(0)}(a/d = 0)$ describes the number of particles channelled through the bent crystal relative to the number of particles channelled through the straight one. Hence, it is convenient to use the quantity $\langle \mathrm{d}E \rangle$ to compare the spectra produced by effectively different number of projectiles as it happens for different a/d values.

Figures 3 correspond to the spectra (5) obtained as outlined above. The results presented were calculated by using the Molière approximation for interplanar potential.

The first graph in figure 3 corresponds to the case of zero amplitude AW (the ratio $a/d = 0$) and, hence, presents the spectral dependence of the ordinary channelling radiation only. The asymmetric shape of the calculated ordinary channelling radiation peak, which is due to the strong anharmonic character of the Molière potential, bears close resemblance with the experimentally measured spectra [21]. The spectrum starts at $\hbar\omega \approx 960$ keV, reaches its maximum value at 1190 keV, and steeply cuts off at 1200 keV. This peak corresponds to the radiation into the first harmonic of the ordinary channelling radiation (see e.g. [22]), and there is almost no radiation into higher harmonics. The latter fact is consistent with general theory of dipole radiation by ultra-relativistic particles undergoing quasi-periodic motion (see e.g. [12]). Dipole approximation is valid provided the corresponding undulator parameter $p_c = 2\pi\gamma(a_c/\lambda_c)$ is much less than 1. In this relation a_c and λ_c stand for the characteristic scales of, correspondingly, the amplitude and the wave-length of the quasi-periodic trajectory. For the channelling motion one can estimate a_c as $d/2$, and $\lambda_c = c\tau_c$, where $\tau_c \sim 2\pi\sqrt{m\gamma/U''}$ standing for the characteristic period of the channelling oscillations (using the harmonic approximation for the interplanar potential one gets $U'' \sim 8\,U_0/d^2$ where U_0 is the depth of the interplanar potential well). In the case of 0.5 GeV positron channeled along (110) planes in Si one has $\gamma \approx 10^3$, $U_0 = 23$ eV, $d = 1.92$ Å [12]. Hence, $p_c \approx 0.2 \ll 1$ and all the channelling radiation is concentrated within some interval in the vicinity of the energy of the first harmonic. The latter one can estimate as (see e.g. [12]) $\omega_c^{(1)} \sim 4\pi\gamma^2/\tau_c \sim 4\gamma^2 c/d\sqrt{U_0/\varepsilon}$ arriving at the value $\hbar\omega_c^{(1)} \approx 1190$ keV which is exactly the calculated maximum value 1190 keV. More accurate estimates (the details are presented in [10]), based on the account for the dependence of the channeling oscillation period τ_c on the amplitude a_c of the oscillations, also reproduce the calculated position and the width of the peak of the channeling radiation.

Increasing the a/d ratio leads to the modifications in the spectrum of radiation. The changes which occur manifest themselves via three main features, (i) the lowering of the ordinary channelling radiation peak, (ii) the gradual increase of the intensity of undulator radiation due to the crystal bending, (iii) the appearing of additional structure (the sub-peaks) in the vicinity of the first harmonic of the ordinary channelling radiation. Let us discuss all these features.

The decrease in the intensity of the ordinary channelling radiation with the increase of the a/d ratio is related to the simple fact that the growth of the AW amplitude leads to lowering of the allowed maximum value of the channelling oscillations amplitude a_c (this are measured with respect to the centerline of the bent channel) [7]. Indeed, inside the bent channel the motion of the particle is subject to the action of the effective potential

$$U_{eff}(\rho) = U(\rho) - \frac{\varepsilon}{R(z)}\,\rho \tag{6}$$

where $\rho \in [-d/2, d/2]$ is the (local) distance from the centerline and $R(z)$ is the (local) curvature radius of the channel. For a channel bent by a transverse harmonic AW $R = [(\lambda/2\pi)^2/a]\,\sin(2\pi z/\lambda)$. The particle could be trapped into the channelling mode provided its total energy associated with the transverse motion is less the minimal value, $U_{eff}(\rho_0)$, of the two maxima points of the asymmetric potential well described by (6) [7, 16]. The potential $U_{eff}(\rho)$ reaches the magnitude of $U_{eff}(\rho_0)$ at some point ρ_0 which satisfies the condition $|\rho_0| < d/2$ and the absolute value of ρ_0 decreases with the growth of a. Hence, the larger the channel is bent the lower the allowed values of the channelling oscillations amplitude are, and, consequently, the less intensive is the channelling radiation, which is, essentially, proportional to the squared amplitude [12]. Let us note, that since we have restricted the range of the a_c values by imposing the condition $a_c < d/2 - a_{TF}$ (see the discussion above (5)), then the decrease of the intensity of channelling radiation occurs starting with some non-zero value of the AW amplitude for which $|\rho_0|$ also becomes less than $d/2 - a_{TF}$.

These arguments explain the fact that the intensities of the ordinary channelling radiation for $a/d = 0$ and $a/d = 1$ are much alike, while for $a/d > 1$ it starts loosing the magnitude.

The undulator radiation (the AIR) related to the motion of the particle along the centerline of periodically bent channel is absent in the case of straight channel (the graph $a/d = 0$), and is almost invisible for comparatively small amplitudes of the AW (see the graph for $a/d = 1$). With a increasing the peaks corresponding to AIR are becoming more prominent, and for large a values ($a/d \sim 10$) two additional features appear: the intensity of the first harmonic of the AIR becomes larger than the intensity of the ordinary channelling radiation, and there appears radiation into the third harmonic of the AIR.

The positions and the widths of the AIR peaks can be quite accurately estimated as follows (see [1, 2]): $\omega^{(n)} = 8\pi\gamma^2 c\lambda^{-1}\, n/(2+p^2)$, where the integer $n = 1, 2 \ldots$ enumerates the harmonics, and $p = 2\pi\gamma(a/\lambda)$ stands for the parameter of the undulator related to the periodicity of the channel bending. The width of each peak $\Delta\omega = (1/N)(\omega^{(n)}/n)$ is independent on n. Substituting the values of γ, λ and d indicated above one expresses the undulator parameter via the ratio a/d: $p \approx 0.05(a/d)$. Even for the largest considered value $a/d = 10$ the parameter p is less than 1, thus making the radiation into higher harmonics of the AIR almost negligible compared with the intensity radiated into the fundamental harmonic $n = 1$. The latter is located at $\hbar\omega^{(1)} \approx 90$ keV having the width $\hbar\Delta\omega \approx 6$ keV which is almost 40 times less than the width of the peak of the channeling radiation. These values depend neither on the ratio $a/d = 10$ nor on the type of the interplanar potential.

As mentioned, all graphs in figure 3 refer to the forward emission. Therefore, in accordance with general theory of the undulator radiation (see e.g. [12]), the second peak of the AIR, which is mostly pronounced in the case $a/d = 10$, corresponds to the third harmonic of AIR, and is located at $\hbar\omega^{(2)} \approx 270$ keV. The intensities radiated into the fundamental and the third harmonics are equal to 1.10×10^5 sr^{-1} and 7.12×10^3 sr^{-1}, respectively. Their ratio is approximately equal to p^{-4} which is also in accordance with general theory.

The intensity of the AIR gradually increases with the AW amplitude. More precisely, $\mathrm{d}E_{\mathrm{AIR}} \propto p^2 \propto (a/d)^2$ (in the case $p < 1$), and this tendency one can observe comparing the AIR peaks in the graphs corresponding to $a/d = 4, 6, 8, 10$.

It is important to note that the positions of sharp AIR peaks, their narrow widths, and the radiated intensity are, practically, insensitive to the choice of the approximation used to describe the interplanar potential. In addition, provided the condition (2) is fulfilled, the AIR peaks are well separated (in the photon energy scale) from the peaks of the channeling radiation. Therefore, if being interested in the spectral distribution of the AIR only, one may disregard the channeling oscillations and to assume that the projectile moves along the centerline of the bent channel [1, 2]. The above statements are illustrated by figure 4 where we compare the results of calculations of the total spectrum (5) in vicinity of the first harmonic of AIR in the case $a/d = 10$. All parameters are the same as in figure 3. The filled and open circles represent the results of evaluation of the right-hand sides of (2) and (5) accompanied by numerical solution of the equations of motion for the projectile within the Molière (filled circles) and the harmonic (open circles) approximations for the interplanar potential. The solid line corresponds to the AIR radiation only (in this case the numerical procedures are simplified considerably, leading to the reduction, by orders of magnitude, of the CPU time). It is clearly seen that the more sophisticated treatment almost does not change the profile of the peak obtained by means of simple formulae describing purely AIR radiation [1, 2]. Moreover, the minor changes in the position and the height of the peak can be easily accounted for within the framework of the cited formalism by introducing the effective undulator parameter [7, 10] and (in the case of the harmonic approximation) the effective undulator amplitude [10].

Thus, in vicinity of the AIR peaks there is no coupling of the two mechanisms of the radiation. On the contrary, the AIR strongly affects the spectrum in the photon energy range corresponding to the channeling radiation. The discussion of this effect lies beyond the scope of this Letter and is presented in [10]. Here we only mention that, as it is clearly seen from the graphs $a/d = 6, 8, 10$ in figure 3, for large values of a/d the peaks of the ordinary channeling radiation acquire additional structure: there appear sub-peaks separated (in the case of the forward emission) by the interval $\delta\omega = 2\omega^{(1)}$.

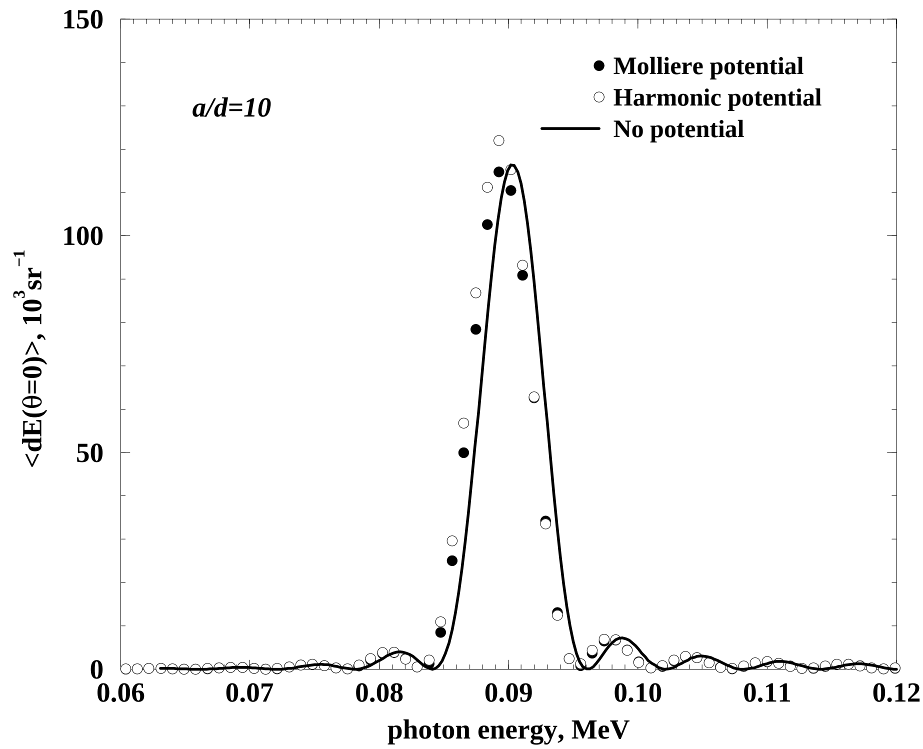

Figure 4. Comparison of different approximations used to calculate the total ardiative spectrum in vicinity of the first harmonic of the AIR. The ratio $a/d = 10$, other parameters as in figure 3. See also the commentaries in the text.

The presented results of the numerical calculations of the total spectrum of radiation formed in an acoustically bent crystal clearly demonstrate the validity of the statement made in [1, 2, 7] that the AIR and the ordinary channeling radiation occur in essentially different ranges of the emitted photons energies, allowing, thus, to investigate the AIR properties separately from the ordinary channeling radiation.

The positions of the peaks of the AIR as well as their intensity can be easily varied by changing the energy of projectile, the crystal type, the type of crystallographic plane, and the parameters of periodic pattern of crystal bendings. This statement by no means is restricted to the case of the acoustically bent crystal, but has more general nature. The treatment and the main results presented above can be applied, in particular, to consider the undulator radiation in statically bent crystals [2, 3]. Whatever way is chosen to prepare periodically bent crystalline lattice it is worth investigating, both theoretically and experimentally, not only the spontaneous emission of the AIR-type, but the stimulated emission as well [2].

Acknowledgments

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